**Smith** **Math 290** **Spring 2010**

**Exam** 3

I affirm this work abides by the university's Academic Honesty Policy.

________________________________________

**Print Name, then Sign**

**Directions:**

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

---

## Do any two (2) of the following of these "Computational" problems

**C.1.** [15 points] Show that the set $W = \left\{p \in P^4 \mid p'' + 2p' + p = z\right\}$ is a subspace of $P^4$. Here, $z$ is the polynomial $z(t) = 0t^4 + 0t^3 + 0t^2 + 0t + 0$.

**C.2.** [15 points] The matrix $A = \begin{bmatrix} 25 & -8 & 30 \\ 24 & -7 & 30 \\ -12 & 4 & -14 \end{bmatrix}$ has $\lambda = 1$ as an eigenvalue. Find a basis for the eigenspace corresponding to this eigenvalue.

**C.3.** [15 points] What is the dimension of the subspace of $P^4$ spanned by $T = \left\{x^3 - 3x + 1,\ x^4 - 6x + 3,\ x^4 - 2x^3 + 1\right.$

**C.4.** [7, 8 points] Given $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

- 1. Prove that $p_A(x) = x^2 - Trace(A)\,x + \det(A)$
  2. If $\lambda_1, \lambda_2$ are the eigenvalues of $A$, then $Trace(A) = \lambda_1 + \lambda_2$ and $\det(A) = \lambda_1\lambda_2$.

## Do any two (2) of these "In Class, Text, or Homework" problems

**M.1.** [15 points] Given a square matrix $A$ satisfying $A^3 = A$ show that the only possible eigenvalues for $A$ are 0 and $\pm 1$.

**M.2.** [15 points] Suppose that $A$ is an $m \times n$ matrix and that $\vec{b} \in \mathbf{C}^m$. Prove that the linear system $LS\left(A, \vec{b}\right)$ is consistent if and only if $r(A) = r\left(\left[A \mid \vec{b}\right]\right)$.

**M.3.** [15 points] Let $A$ be a particular square matrix of size $n$. Show that the set $W = \{B \in M_{nn} \mid AB = BA\}$ is a subspace of $M_{nn}$.

**Do any two (2) of these "Other" problems**

**T.1.** [15 points] Let $V$ be a vector space and $S$ a subset of $V$. Prove that $S$ is a subspace of $V$ if and only if $S = \langle S \rangle$.

**T.2.** [15 points] If $A$ and $B$ are both $n \times n$ matrices and $B$ is nonsingular, prove that if $\lambda$ is an eigenvalue of the product matrix $C = AB$, then $\lambda$ is also an eigenvalue of $D = BA$.[Hint: what would the corresponding eigenvectors look like?]

**T.3.** [15 points] Extend the result in problem T.2. above by proving that $\lambda$ is an eigenvalue of $D = BA$ even if $B$ is singular.

**T.4.** [15 points] Given a basis $\{\vec{v}_1, \cdots, \vec{v}_n\}$ of $\mathbf{C}^n$ and a matrix $A$ for which $\{\vec{v}_{r+1}, \cdots, \vec{v}_n\}$ is a basis for the null space of $A$. Prove **one** (1) of the following

1. The set $\{A\vec{v}_1, \cdots, A\vec{v}_r\}$ spans the column space of $A$.
2. The set $\{A\vec{v}_1, \cdots, A\vec{v}_r\}$ is linearly independent in $\mathbf{C}^n$.